



Oxford Cambridge and RSA

Friday 20 June 2025 – Afternoon

A Level Further Mathematics B (MEI)

Y435/01 Extra Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

QP

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

- 1 A binary operation, $\circ$, is defined on real numbers a and b by $a \circ b = 3ab$.
- (a) Prove that $\circ$ is associative over the real numbers $\mathbb{R}$. [2]
 - (b) Determine the value of e , the identity element for $\circ$. [2]
 - (c) Explain why $\mathbb{R}$ is closed under $\circ$. [1]
 - (d) Explain why $\mathbb{R}$ under $\circ$ does **not** form a group. Justify your answer. [2]
- 2 (a) Determine the general solution of the recurrence relation $25u_{n+2} - 35u_{n+1} + 12u_n = c$, $n \geq 0$, where c is a constant. [5]

The manager of a company is using the recurrence relation in part (a) to model the number of containers held by the company using the sequence $u_0, u_1, u_2, \dots$ which satisfies this recurrence relation.

The quantity u_0 represents the number of containers held by the company at the **start** of January 2000. The quantity u_n , for $n \geq 1$, represents the number of containers held by the company at the **end** of month n , where the first month is January 2000. If necessary, the quantity u_n is rounded to the nearest integer.

The company holds 20 containers at the start of January 2000 and 70 containers at the end of January 2000.

- (b) In an initial model the manager uses $c = -10$.
 - (i) Determine the particular solution for u_n when $c = -10$. [3]
 - (ii) The model predicts a single peak value for u_n .
Find, by direct calculation, the largest number of containers held by the company according to the model. [2]
 - (iii) Show that the model in part (b)(i) breaks down in the 19th month. [3]
- (c) In a second model the manager uses $c = 10$ instead of $c = -10$.
Show that the second model predicts that eventually the number of containers held by the company will be a constant k , where k is to be determined. [2]

3

- 3 M is the group $(\{0, 1, 2, 3, 4, 5, 6, 7, 8\}, +_9)$ where $+_9$ denotes the binary operation of addition modulo 9.

(a) Write down the identity element of M . Justify your answer. [1]

(b) By considering the order of M , explain why there can be **no** subgroup of M of order 5. [2]

You are given that M is a cyclic group.

(c) Find **all** possible generators of M . [2]

You are given that there is only one proper, non-trivial subgroup of M , denoted by $(H, +_9)$.

(d) (i) Write down H . [1]

(ii) Show that there can be **no** other proper, non-trivial subgroups of M . [2]

- 4 A surface, S , is defined in 3-D by $z = f(x, y)$ where $f(x, y) = x^3 - 12xy^2 + 96y^2 + 30$.

You are given that the point $(0, 0, 30)$ on S is a stationary point.

(a) Determine the coordinates of any other stationary points on S . [6]

(b) By sketching the section $z = f(x, 0)$ on the coordinate axes provided in the Printed Answer Booklet, determine the nature of the stationary point $(0, 0, 30)$ on S . [2]

The normal to S at the point where $x = 3$ and $y = 2$ passes through the point $(a, 482, 295)$ where a is a constant.

(c) Determine the value of a . [5]

The contour given by $z = 542$ comprises a straight line and a curve. The curved portion of the contour is denoted by C . You are given that there are **two** points on C where the x coordinate is 8.

(d) Determine the values of the y coordinates at these **two** points on C . [3]

Turn over for question 5

5 In this question you must show detailed reasoning.

The matrix $\mathbf{A}$ is given by $\begin{pmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & 1 \end{pmatrix}$ where a and b are constants with $a^2 + b^2 = 1$ and $a \neq \pm 1$.

- (a) (i) By finding the characteristic equation of $\mathbf{A}$ in terms of a , show that, for any value of a , 1 is always one of the eigenvalues of $\mathbf{A}$. [3]
- (ii) By showing that the other eigenvalues of $\mathbf{A}$ are $a \pm \sqrt{a^2 - 1}$, prove that the other eigenvalues of $\mathbf{A}$ are **not** 1. [3]
- (b) Hence show that **either** some of the eigenvalues of $\mathbf{A}$ must be non-real **or** some of the elements of $\mathbf{A}$ must be non-real. [3]

You are now given the following information.

- a and b are both real.
 - $\mathbf{A}$ represents a rotation by an acute angle in 3-D space.
 - $\mathbf{e}$ is a real eigenvector of $\mathbf{A}$ associated with the eigenvalue 1.
- (c) (i) Explain the significance of $\mathbf{e}$, and the fact that its associated eigenvalue is 1, in relation to the transformation that $\mathbf{A}$ represents. [2]
- (ii) Explain **geometrically** why the other eigenvectors of $\mathbf{A}$ can **not** be real. [1]
- (iii) Find a vector equation for the axis of rotation of the transformation that $\mathbf{A}$ represents. [2]

END OF QUESTION PAPER

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